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CALIFORNIA UNIV SANTA CRUZ DEPT OF MATHEMATICS
J GUCKENHEIMER AUG 84 AFOSR-TR-84-0914 AFOSR-83-0143
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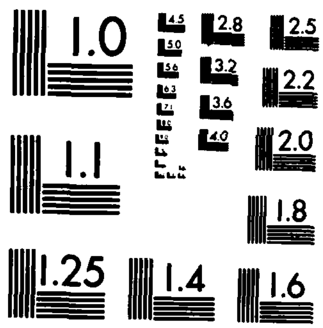
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ANNUAL TECHNICAL REPORT

JOHN GUCKENHEIMER

DYNAMICAL CHARACTERISTICS OF WEAK TURBULENCE

AFOSR-83-0143

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UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 84-0914
6a. NAME OF PERFORMING ORGANIZATION University of California / 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State, and ZIP Code) Department of Mathematics Santa Cruz CA 95064	7b. ADDRESS (City, State, and ZIP Code) Directorate of Mathematical & Information Sciences, AFOSR, Bolling AFB DC 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR / 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-83-0143
8c. ADDRESS (City, State, and ZIP Code) Bolling AFB DC 20332	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61102F / PROJECT NO. 2304 / TASK NO. A4 / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
DYNAMICAL CHARACTERISTICS OF WEAK TURBULENCE

12. PERSONAL AUTHOR(S)
John Guckenheimer

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Interim	FROM 1/4/83 TO 31/3/84	AUG 84	13

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This project is an exploration of dynamical features of chaotic physical systems with the emphasis upon turbulent fluids. The specific areas of investigation involve (1) the development of techniques that discriminate measurable differences in the observed behavior of theoretical models for chaotic behavior, (2) the application of these techniques to experimental studies, and (3) the study of bifurcation behavior in multiparameter families of differential equations.

The transition to chaotic behavior in fluids has received intense experimental study during the past ten years. Various "routes to chaos" have been studied, and a satisfying picture has emerged of how this transition proceeds in low dimensional dynamical systems. The author's primary interest is in the behavior of a system after it has undergone the transition. Of central concern is the question of determining when low dimensional chaotic models provide a good description of the physical system. (CONTINUED)

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Robert N. Buchal	22b. TELEPHONE (Include Area Code) (202) 767- 4939 / 22c. OFFICE SYMBOL NM

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

84 10 23 195

ITEM #19, ABSTRACT, CONTINUED: Considerable progress has been made in the development of efficient algorithms that estimate a fractal dimension of attractors. The work supported by this grant includes one of the first applications of these methods to data from experimental studies. These studies reveal that the dimension of the attractors describing the fluid motion in its state space grow rather quickly with parameters. This fact poses a considerable challenge for future investigation. When a system has an attractor of even moderate dimension, it is no longer feasbiel to observe it long enough to obtain a detailed picture of the attractor. Using the techniques that have gone towards the work done thus far, one may have to be content with tools that discriminate regimes describable by low dimensional chaotic models from others.

The second major area of investigation involves bifurcation theory for differential equations. Here the goal is to classify, insofar as possible, the way in which the qualitative dynamics of a system depend upon parameters. Current work in this direction involves studying the role of symmetries upon the bifurcations and work with models of chemical reactors.

ABSTRACT

DYNAMICAL CHARACTERISTICS OF WEAK TURBULENCE

This project is an exploration of dynamical features of chaotic physical systems with the emphasis upon turbulent fluids. The specific areas of investigation involve (1) the development of techniques that discriminate measurable differences in the observed behavior of theoretical models for chaotic behavior, (2) the application of these techniques to experimental studies, and (3) the study of bifurcation behavior in multiparameter families of differential equations.

The transition to chaotic behavior in fluids has received intense experimental study during the past ten years. Various "routes to chaos" have been studied, and a satisfying picture has emerged of how this transition proceeds in low dimensional dynamical systems. Our primary interest is in the behavior of a system after it has undergone the transition. Of central concern is the question of determining when low dimensional chaotic models provide a good description of the physical system.

Considerable progress has been made in the development of efficient algorithms that estimate a fractal dimension of attractors. The work supported by this grant includes one of the first applications of these methods to data from experimental studies. These studies reveal that the dimension of the attractors describing the fluid motion in its state space grow rather quickly with parameters. This fact poses a considerable challenge for future investigation. When a system has an attractor of even moderate dimension, it is no longer feasible to observe it long enough to obtain a detailed picture of the attractor. Using the techniques that have gone towards the work done thus far, one may have to be content with tools that discriminate regimes describable by low dimensional chaotic models from others.

The second major area of investigation involves bifurcation theory for differential equations. Here the goal is to classify, insofar as possible, the way in which the qualitative dynamics of a system depend upon parameters. Current work in this direction involves studying the role of symmetries upon the bifurcations and work with models of chemical reactors.

DYNAMICAL CHARACTERISTICS OF WEAK TURBULENCE

Research Objectives

The overall goal of this research is the development of qualitative and geometric tools for studying the dynamics of physical systems - particularly when the dynamics are complex. Attention has been especially directed at fluid systems in regimes near the transition to turbulence. There are two specific areas of investigation that have been emphasized:

(1) The classification of typical bifurcations in multi-parameter families of differential equations and in families of differential equations with a specified symmetry.

(2) The analysis of aperiodic data with a view towards determining whether the data can be modelled by a strange attractor.

These goals bear upon applications in the following manner. Much effort has been directed towards the experimental study of the "routes to chaos" in fluid systems. Different sequences of transitions are observed in different systems and one would like a theory which could predict the results of such experiments. The bifurcation theory problems being studied are directed towards this goal. The regions of a parameter space near the coincidence of different types of bifurcation are tractable places in which to calculate successive bifurcations and to study the effect of nonlinear interactions between different modes which are becoming unstable. The theory developed here should have broad usefullness well beyond the applications to fluid systems. For example, multiple bifurcation theory is a promising tool for studying complicated dynamical behavior of chemical reaction mechanisms. These give rise to systems of differential equations which are usually time consuming to integrate numerically (due to different time scales of component reactions) and have many parameters.

The work on data analysis is motivated by the desire to model aperiodic behavior by systems with few degrees of freedom when this is possible. Since fluid systems have an infinite number of degrees of freedom, one would like to determine those regimes in which reduced models are possible. The "traditional" technique of identifying chaotic behavior with a continuous power spectrum does not distinguish data which is random, has many degrees of freedom, or comes from a strange attractor with few degrees of freedom. This has been the intent of the data analysis sponsored by this project.

Status of the Research

There has been significant progress in achieving the research objectives described above. We describe the situation with regard to data analysis first.

An attractor of a dynamical system is a set of trajectories towards which other trajectories are drawn (and does not contain smaller sets with this property). An additional concept of relevance to physical applications is the idea of an asymptotic measure for the attractor. This is defined as a measure with the property that it describes the time that most trajectories in the attractor spend in different regions of the attractor. If one thinks of a moving point in phase space which leaves a uniform trail of ink, the asymptotic measure describes the expected relative density of ink after a long time. With appropriate technical assumptions, one can define a "fractal" dimension for a measure. The most natural definition of this dimension for the applications is that it is given by the scaling behavior of the measure of balls. In a space of dimension d, the measure of balls of radius r will be proportional to r^d as $r \to 0$.

The fractal dimension of an attractor (or its asymptotic measure) provides a lower bound for the number of degrees of freedom which one needs to model the attractor. Therefore, estimates of the fractal dimension of an attractor obtained from a trajectory are directly relevant to the questions posed above.

Research sponsored by this project has developed an efficient experimental technique for estimating the dimension of (the asymptotic measure of) an attractor from a trajectory. The technique is based upon using interpoint distances between observations to estimate the volume of balls with respect to the asymptotic measure. Similar ideas have been used by Procaccia and coworkers and by Farmer and coworkers. One of the first applications of this technique is the paper by Guckenheimer and Buzyna, examining data from experiments measuring low in a rotating annulus. This technique has the potential for becoming a standard diagnostic tool for distinguishing data describable in terms of low dimensional strange attractors from more complicated aperiodicity. Further refinement of the technique is needed as well as more extensive investigation of the dimensional properties of specific examples like the Henon attractor.

There has also been progress on multiple bifurcation problems. A systemic review of codimensions two bifurcations at equilibria appeared in January 1984. More recent work has focussed upon the analysis of problems with symmetry, upon problems of higher codimension, and upon examples arising in the theory of chemical reactors. A review of the applications to chemical reactor theory is being prepared, and it will contain a number of new results about model equations for stirred tank

reactors. Current work is underway on the effect of symmetry breaking perturbations of a problem with circular symmetry. Examples of such a problem are the motion of a horizontally forced sperical pendulum (previously studied by Miles) and the sloshing of a cylindrical container of fluid (studied by Sethna). It is our feeling that the previous work in the bifurcation behavior in these problems is incomplete and that an approach which focuses upon generic aspects of the problems is warranted.

List of Publications

John Guckenheimer

1) Endomorphisms of the Riemann Sphere, Proc. Am. Math. Soc., Symposia in Pure Math. XIV: 95-124, 1970.

2) Holomorphic Vector Fields on CP(2) , An. Acad. Brasil. Cienc. 42: 415-420, 1970.

3) Axiom A + No Cycles implies $zetz_f(t)$ Rational. Bulletin Am. Math. Soc. 76: 592-594, 1970.

4) Hartman's Theorem for Complex Flows in the Poincare Domain. Compositio, 24: 75-82, 1972.

5) Absolutely Omega-Stable Diffeomorphisms, Topology, 11: 195-197, 1972.

6) Bifurcation and Catastrophe, Dynamical Systems, ed. by M.M. Peixoto, Academic Press, pp. 99-110, 1973.

7) One Parameter Families of Vector Fields: Another Non-Density Theorem, Dynamical Systems, ed. by M.M. Peixoto, Academic Press, pp. 111-128, 1973.

8) Catastrophes and Partial Differential Equations. Annales de l'Institut Fourier 23: 31-59, 1973.

9) Review of Stabilite Structurelle et Morphogenese by Rene Thom, Bulletin Am. Math. Soc. 79: 878-890, 1973.

10) Caustics, Global Analysis and its Applications. V. II, International Atomic Energy Agency, pp. 281-289, 1974.

11) Caustics and Non-degenerate Hamiltonians. Topology 13: 127-133, 1974.

12) Solving a Single Conservation Law, Warwick Dynamical Systems, 1974, Springer Lecture Notes in Mathematics, pp. 108-134.

13) Isochrons and Phaseless Sets. Journal of Mathematical Biology 1: 259-273, 1975. (Reprinted also in Studies in Mathematical Biology, Part I, Mathematical Association of America.)

14) Shocks and Rarefactions in Two Space Dimensions. Arch. Rational Mec. Analysis 59: 281-291, 1975.

15) G. Oster and ______, Bifurcation Phenomena in Population Models, in The Hopf Bifurcation Theorem and its Applications, ed. J. Marsden and M. McCracken, Springer-Verlag, 1976, pp. 327-353.

16) A Strange, Strange Attractor, in The Hopf Bifurcation Theorem and its Applications, ed. J. Marsden and M. McCracken, Springer-Verlag, 1976, pp. 368-381.

17) ______, G. Oster, and A. Ipaktchi, Dynamics of Deterministic Population Models, J. Math. Biology 4, pp. 101-147, 1977.

18) Constant Velocity Waves in Oscillating Chemical Reactions, in Structural Stability, Theory of Catastrophes and Applications, Springer-Verlag, 1976, pp. 99-103.

19) The Cusps of Zeeman's Catastrophe Machine, Topology, 16, pp. 177-180, 1977.

20) D. Auslander, ______, and G. Oster, Random Evolutionarily Stable Strategies, Journal of Theoretical Population Biology, 13, pp. 276-293, 1978.

21) On the Bifurcation of Maps of the Interval, Inventiones Math. 39, 165-178, 1977.

22) Comments on Catastrophe and Chaos, to appear in Some Mathematical Questions in Biology VIII, Am. Math. Soc., 1977.

23) ______ and R.F. Williams, Structural Stability of Lorenz Attractors, to appear in Publ. I.H.E.S., no. 50, 60-72, 1979.

24) The Bifurcation of Quadratic Functions, Annals N.Y. Acad. Sci., 316 pp. 78-85, 1979.

25) The Catastrophe Controversy, Mathematical Intelligencer, 1, pp. 15-20, 1978.

26) ____________, Jurgen Moser, Sheldon Newhouse, Dynamical Systems, CIME Lectures, Bressanone, 1978. Birkhauser, 1980, 289 pgs.

27) A Brief Introduction to Dynamical Systems, Am. Math. Soc. - SIAM series, Lectures in Applied Mathematics, v. 17, pp. 187-253, 1979.

28) Instabilities and Chaos in Nonhydrodynamic Systems, chapter 9 of Hydrodynamic Instabilities and the Transition to Turbulence, ed. by H.L. Swinney, pp. 271-287, Springer-Verlag, 1980.

29) Sensitive Dependence to Initial Conditions for One Dimensional Maps, Comm. Math. Phys. 70, pp. 133-160, 1979.

30) Symbolic Dynamics and Relaxation Oscillations, Physica D, 1, 1980, 227-235.

31) On a Codimension Two Bifurcation, Dynamical Systems and Turbulence, Warwick 1980 (ed. D. Rand and L.-S. Young) Lecture Notes Math 898, pp. 99-142, 1981.

32) Growth of Topological Entropy for One Dimensional Maps, Global Theory of Dynamical Systems, Proceedings, Northwestern, 1979, Lecture Notes in Math. 819, 1980, pp. 216-223.

33) L. Block, ____________, M. Misiurewicz, Lai Sang Young, Periodic Points of One Dimensional Maps, ibid, pp. 18-34.

34) Review of Catastrophe Theory and its Applications by T. Poston and I. Stewart, SIAM Review, 21, pp. 572-573, 1979.

35) Patterns of Bifurcation, New Approaches to Nonlinear Problems in Dynamics, SIAM, 1980, pp. 71-104.

36) D. Brillinger, ____________, P. Guttorp, G. Oster, Empirical Modelling of Population Time Series Data, Lectures on Mathematics in the Life Sciences, vol. 13, pp. 65-90, 1980.

37) One Dimensional Dynamics, N.Y. Acad. Sci., v. 316, pp. 76-85, 1979.

38) Dynamics of the Van der Pol Equation, IEEE, Trans. Circuits and Systems 27, 983-989, 1980.

39) ____________, E. Knobloch. Nonlinear Convectic in a Rotating Layer: Amplitude Expansions and Norm. rms, Geophysical and Astrophysical Fluid Dynamics, 23, pp. 247-272, 1983

40) E. Knobloch, ____________. Convective Transitions Induced by Varying Aspect Ratio, Physical Review A, 27, pp.408-417, 1983.

41) An Introduction to Chaotic Motion and Strange Attractors, Lectures in Applied Mathematics, Am. Math. Soc., 20, pp.220-227, 19

42) Noise in Chaotic Systems, Nature, 298, pp. 358-361, 1982.

43) Persistent Properties of Bifurcations, LANL Conference on Order in Chaos, Physica 7D, 105-110, 1983.

44) Strange Attractors in Fluid Dynamics, Sitges Conference on Dynamical Systems and Chaos, Lecture Notes in Physics, 179, Springer-Verlag, pp.149-157, 1983.

45) Multiple Bifurcations of Codimension Two, SIAM J. Math. Anal., Vol. 15, No. 1, January 1984.

46) ____________, Philip Holmes. Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, 1983, 453 pp.

47) Toolkit for Nonlinear Dynamics, IEEE Trans. Circuits and Systems, August 1983, Vol. CAS-30, No. 8, pp. 586-590.

48) ____________, George Buzyna. Dimension measurements for Geostrophic Turbulence, Phys. Rev. Letters, 51, 1438-1441, 1983.

49) Dimension estimates for attractors, Contemporary Mathematics, 28, 357-367, 1984.

50) Clues to Strange Attractors, Proceedings, NATO Workshop on Chaos in Astrophysics, April 1984.

51) ____________, Richard McGehee. A Proof of the Mandelbrot N^2 Conjecture, preprint, Institut Mittag-Leffler, 1984.

52) Multiple Bifurcation Problems for Chemical Reactors. Preprint, Santa Cruz, 1984.

PERSONNEL

John Guckenheimer	Ph.D., Mathematics, 1970
Bob Lansdon	B.A., Mathematics, 1976

SEMINARS AND COLLOQUIA

John Guckenheimer

1981

Theoretical Physics Institute, Santa Barbara, October 9

U.S. - Japan Workshop on Chaos and Plasma Physics, Kyoto, Japan, November 9 - 13.

Mathematics Department, Kyoto University, November 14

1982

Los Alamos Conference on Order in Chaos, May 24-28

Singularity Theory Seminar, U.C. Berkeley, June 2

Geophysical Fluid Dynamics Institute, Florida State University, June 7 - 11 (two lectures)

AMS Summer Research Conference, June 14 - 18

Theoretical Physics Institute, Santa Barbara, July 30

Sitges (Spain) Conference on Theoretical Physics, September 13 - 17

University of Chicago, Mathematics and Physics Departments (3)

University of California, Berkeley (2)

1983

University of California, Berkeley, Physics Department

Los Alamos Center for Nonlinear Studies

University of Maryland, Symposium on Chaos

Western States Mathematical Physics Meeting, Calif. Inst. Tech.

Calif. Inst. of Tech., Physics Department

IEEE International Circuits and Systems Conference, Newport Beach

City University of New York, Mathematics Department (2)

1983 (con't)

University of California, Berkeley, Mathematics Department

NATO Workshop on Testing Nonlinear Dynamics, Haverford

American Mathematical Society Research Conference on Fluids and Plasmas, University of Colorado

American Mathematical Society Regional Conference on Rational Maps, University of Minnesota, Duluth

Cornell University, Mathematics Colloquium, October 27

Cornell University, Applied Mathematics Colloquium, October 28

University of Houston, Mathematics Colloquium, November 17

University of Houston, Physics Colloquium, November 18

1984

San Diego, Dynamics Days, January 4

UCSC, Mathematics Colloquium, January 10

UCLA, Mathematics/Nonlinear Studies Colloquium, February 9

UC Berkeley, Nonlinear dynamics seminar, March 21

M.S.R.I., Berkeley, Rational Maps Workshop, March 26

Florida, NATO Workshop on Chaos in Astrophysics, April 11

Mittag-Leffler Institute, Sweden, Colloquium, May 2

Mittag-Leffler Institute, Sweden, Colloquium, May 23

Linkoping, Sweden, Swedish Mathematical Society, May 17

Trondheim, Norway, Mathematical Colloquium, June 13

OTHER PROFESSIONAL ACTIVITIES

Chairman Organizing Committee. Workshop on Turbulence, MSRI, January, 1984

Co-Chairman Organizing Committee, Workshop on Rational Maps, MSRI, March, 1984

Founding Chairman, University of California Coordinating Committee on Nonlinear Studies

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